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Relationship between Quark-Meson Coupling Model and Quantum Hadrodynamics

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Using the quark-meson coupling (QMC) model, we study nuclear matter from the point of view of quark degrees of freedom. Performing a re-definition of the scalar field in matter, we transform QMC to a QHD-type model with a non-linear scalar potential. The potentials obtained from QMC are compared with those of the relativistic mean-field models.

Since at present rigorous studies of quantum chromodynamics (QCD) are limited to matter system with high temperature and zero baryon density, it is important to build models which help to bridge the discrepancy between nuclear phenomenology and QCD. We have proposed a relativistic quark model for nuclear matter and finite nuclei, that consists of non-overlapping nucleon bags bound by the self-consistent exchange of isoscalar, scalar (σ) and vector (ω) mesons in mean-field approximation (MFA) – this model is called the quark-meson coupling (QMC) model.[1)] On the other hand, recent theoretical studies show that various properties of finite nuclei can be very well described by the relativistic mean-field (RMF) models, i.e., Quantum hadrodynamics (QHD).[2)] In this letter, we consider relationship between QMC and QHD and study how the internal structure of the nucleon sheds its effect on effective nuclear models.

In our previous works,[1)] the MIT bag model has been used to describe the quark structure of the nucleon. Since the confined quarks interact with the scalar field, σ, in matter, the effective nucleon mass, $M_N^\star$, in QMC is given by a function of σ through the quark model of the nucleon. (Although the quarks also interact with the ω meson, it has no effect on the nucleon structure except for a shift in the nucleon energy.[1)]) The (relativistic) constituent quark model (CQM) is an alternative model for the nucleon. Recently, Shen and Toki[3)] have proposed a new version of QMC – the quark mean-field (QMF) model, where CQM is used to describe the nucleon.

In the present study, as well as the bag model (BM), we want to use the relativistic CQM with confining potentials, $V(r)$, of a square well (SW) and a harmonic oscillator (HO) to see the dependence of the matter properties on the quark model. It is assumed that the light (u or d) quark mass, m_q, is 300 MeV in CQM, while $m_q = 0$ MeV in BM. Furthermore, we introduce a Lorentz-vector type confining potential, which is proportional to γ_0, as well as the scalar one:

$$V(r) = (1 + \beta\gamma_0)U(r), \tag{1}$$

where the potential, $U(r)$, is given by SW or HO and $\beta(0 \leq \beta < 1)$ is a parameter to control the strength of the Lorentz-vector type potential. We assume that the shape of the Lorentz-vector type confining potential is the same as that of the scalar type one.

In SW, the solution for a quark field, ψ_q, can be calculated *à la Bogolioubov.*[4] The potential is given by $U(r) = 0$ for $r \le R$ and M for $r > R$, where R is the radius of the spherical well and M is the height of the potential outside the well. After finishing all calculations, the limit $M \to \infty$ is taken.[5] This system may be described by Lagrangian density

$$\mathcal{L}_{SW} = \bar{\psi}_q(i\gamma \cdot \partial - m_q)\psi_q \theta(R-r) - \frac{1}{2}\bar{\psi}_q(1+\beta\gamma\cdot a)\psi_q\delta(r-R), \tag{2}$$

where a^μ is the unit vector in time direction: $a^\mu = (1, \vec{0})$. This Lagrangian provides a boundary condition: $i\gamma\cdot n\psi_q = (1+\beta\gamma\cdot a)\psi_q$ at $r = R$, where n^μ is the unit normal outward from the potential surface. This condition gives

$$j_0(x) = \sqrt{\frac{(1-\beta)(E-m_q)}{(1+\beta)(E+m_q)}} j_1(x) \quad \text{at } r = R, \tag{3}$$

with j_n the spherical Bessel function, x the eigenvalue of the confined quark and E the quark energy. (Note that in fact the quark current flows in only the azimuthal direction.[6]) To take into account the corrections of the spurious center of mass (c.m.) motion and gluon fluctuations, we add the familiar form, $-z/R$, with a parameter z for those corrections to the total energy.[4,5] In SW, the nucleon mass, M_N, (at rest) is then given by $M_N = (3\alpha - z)/R$, where $\alpha^2 = x^2 + \xi^2$ and $\xi = Rm_q$.

In case of HO, we set $\beta = 1$ in Eq.(1) because the quark wave function can be obtained analytically.[7] When $U(r) = \frac{1}{2}cr^2$ (c the oscillator strength), a condition to determine the quark energy is given by

$$\sqrt{E+m_q}(E-m_q) = 3\sqrt{c}. \tag{4}$$

The c.m. energy can be evaluated exactly, as in the non-relativistic harmonic oscillator, and it is just one third of the total energy.[7] Thus, the nucleon mass is given by $M_N = 2E - E_g$, where E_g describes gluon fluctuation corrections.[7]

For the MIT bag model there are many good reviews.[5] In BM, we take $m_q = 0$ MeV and $\beta = 0$. (Even in BM it is possible to include the Lorentz-vector type potential using Eqs.(2) and (3). However, if we use a large β in BM, it is hard to get good values of the nuclear matter properties.)

Now we consider an iso-symmetric nuclear matter with Fermi momentum k_F, which is given by $\rho_B = 2k_F^3/3\pi^2$ (ρ_B the nuclear matter density). Then, the total energy per nucleon, E_{tot}, can be written as[1]

$$E_{tot} = \frac{4}{\rho_B(2\pi)^3}\int^{k_F} d\vec{k}\sqrt{M_N^{\star 2}(\sigma) + \vec{k}^2} + \frac{m_\sigma^2}{2\rho_B}\sigma^2 + \frac{g_\omega^2}{2m_\omega^2}\rho_B, \tag{5}$$

where $M_N^\star$ is calculated by the quark model. The σ and ω meson masses, m_σ and m_ω, are taken to be 550 MeV and 783 MeV, respectively. The ω field is determined by baryon number conservation: $\omega = g_\omega\rho_B/m_\omega^2$ (g_ω is the ω-nucleon coupling constant), while the scalar mean-field is given by a self-consistency condition: $(\partial E_{tot}/\partial\sigma) = 0$.[1]

In SW, we set the radius of the potential to be $R = 0.8$ fm and detrmine z so as to fit the free nucleon mass, M_N (= 939 MeV). The parameter β is chosen to be 0 and 0.5 to examine the effect of the Lorentz-vector type confining potential. We find that $z = 4.396$ and 5.164 for $\beta = 0$ and 0.5, respectively. In HO, there are two adjustable parameters, c and E_g. We determine those parameters so as to fit the free nucleon mass and the root-mean-square (charge) radius of the free proton: $r_N^2 = 0.6$ fm^2.[8] (r_N is calculated by the quark wave function.) We find that $c = 1.591$ fm^{-3} and $E_g = 344.7$ MeV for the free nucleon. In nuclear matter, we keep c and E_g constant and the quark energy E varies, depending on the scalar field. In BM, the bag constant, B, and the parameter, z, are fixed to reproduce the free nucleon mass. As in SW, we choose the bag radius of the free nucleon to be 0.8 fm. We find $B^{1/4}$ = 170.3 MeV and $z = 3.273$.[1]

Table I. Coupling constants, $M_N^\star$ and K. The effective nucleon mass, $M_N^\star$, is calculated at ρ_0. The nuclear incompressibility, K, is quoted in MeV. The SW model with β = 0(0.5) is denoted by SW0(5).

	g_σ^2	g_ω^2	$M_N^\star/M_N$	K
SW0	84.4	104	0.725	329
SW5	66.6	65.2	0.807	287
HO	147	64.5	0.805	309
BM	67.6	66.1	0.805	278

Now we are in a position to determine the coupling constants: the σ-nucleon coupling constant, g_σ^2, and g_ω^2 are fixed to fit the nuclear binding energy (-15.7 MeV) at the saturation density ($\rho_0 = 0.15$ fm^{-3}) for nuclear matter. The coupling constants and some calculated properties for matter are listed in Table I. The present quark models can provide good values of the nuclear incompressibility, K.

In SW and BM with *massless* quarks, the quark scalar density in the nucleon[1] vanishes in the limit $\beta \to 1$, which means that the σ meson does not couple to the nucleon.[9] This fact implies that as β is larger the σ-nucleon coupling is weaker in matter. Thus, we can conclude that qualitatively a large mixture of the Lorentz-vector type confining potential leads to a weak scalar mean-field and hence a large effective nucleon mass in nuclear matter. Since in MFA a small effective nucleon mass (and hence a strong scalar field) is favorable to fit various properties of finite nuclei,[2] the confining potential including a strong Lorentz-vector type one may not be suitable for describing a nuclear system.

The main difference between QMC and QHD at *hadronic* level[1] lies in the dependence of the nucleon mass on the scalar field in matter. By performing a re-definition of the scalar field, the QMC Lagrangian density[1] can be cast into a form similar to a QHD-type mean-field model, in which the nucleon mass depends on the scalar field linearly, with self-interactions of the scalar field.[10] In QMC, the nucleon mass in matter is given by a function of σ, $M^\star_{N,QMC}(\sigma)$, through the quark model of the nucleon, while in QHD the mass depends on a scalar field linearly, $M^\star_{N,QHD} = M_N - g_0\phi$ (ϕ is the scalar field in a QHD-type model). Hence, to transform QMC into a QHD-type model, we can apply a re-definition of the scalar field,

$$g_0\phi(\sigma) = M_N - M^\star_{N,QMC}(\sigma), \tag{6}$$

to QMC, where g_0 is a constant chosen so as to normalize the scalar field ϕ in the limit $\sigma \to 0$: $\phi(\sigma) = \sigma + \mathcal{O}(\sigma^2)$. Thus, g_0 is given by $g_0 = -(\partial M^\star_{N,QMC}/\partial\sigma)_{\sigma=0}$. In QMC, we find $g_0 = g_\sigma$ for SW and BM, while $g_0 = \frac{2}{3}g_\sigma$ for HO.

The contribution of the scalar field to the total energy, E_{scl}, is now rewritten in terms of the new field ϕ

$$E_{scl} = \frac{1}{2}\int d\vec{r}\ [(\nabla\sigma)^2 + m_\sigma^2\sigma^2] = \int d\vec{r}\ \left[\frac{1}{2}h(\phi)^2(\nabla\phi)^2 + U_s(\phi)\right], \tag{7}$$

where U_s describes the self-interactions of the scalar field

$$U_s(\phi) = \frac{1}{2}m_\sigma^2\sigma(\phi)^2 \ \text{ and } \ h(\phi) = \left(\frac{\partial\sigma}{\partial\phi}\right) = \frac{1}{m_\sigma\sqrt{2U_s(\phi)}}\left(\frac{\partial U_s(\phi)}{\partial\phi}\right). \tag{8}$$

Note that in uniformly distributed nuclear matter the derivative term in E_{scl} does not contribute. (The effect of this term on the properties of finite nuclei has been studied in Ref. [10].) Now QMC can be re-formulated in terms of the new scalar field, ϕ, and it is of the same form as QHD with the non-linear scalar potential, $U_s(\phi)$, and the coupling, $h(\phi)$, to the gradient of the scalar field. (Note that since this re-definition of the scalar field does not concern the vector interaction, the energy of the ω field (see Eq.(5)) is not modified.)

The Zimanyi-Moszkowski (ZM) model [11] is a good example. By re-definig the scalar field, ZM can be *exactly* transformed to a QHD-type model with a non-linear potential. Since the effective nucleon mass in ZM is given by [11]

$$M^\star_{N,ZM} = \frac{M_N}{1 + (g_\sigma\sigma/M_N)}, \tag{9}$$

the model involves higher order couplings between the σ and the nucleon. Introducing a new scalar field ϕ by $g_0\phi(\sigma) = M_N - M^\star_{N,ZM}(\sigma)$, we easily find

$$g_0 = g_\sigma, \ \ \phi(\sigma) = \frac{\sigma}{1+g'\sigma} \ \text{ and } \ \sigma(\phi) = \frac{\phi}{1-g'\phi}, \tag{10}$$

with $g' = g_\sigma/M_N$. Thus, the non-linear potential is given by Eq.(8)

$$U_s(\phi) = \frac{1}{2}m_\sigma^2\left(\frac{\phi}{1-g'\phi}\right)^2. \tag{11}$$

In general, the in-medium nucleon mass may be given by a complicated function of the scalar field. However, in QMC the mass can be parametrized by a simple expression up to $\mathcal{O}(g_\sigma^2)$: [1]

$$M^\star_N/M_N \simeq 1 - ay + by^2, \tag{12}$$

with $y(= g_\sigma\sigma/M_N)$ a dimensionless scale and two (dimensionless) parameters, a and b. This parametrization is accurate up to $\sim 4\rho_0$. [1]

Once the parameters, a and b, are fixed, we can easily re-define the scalar field using Eq.(6). We find

$$g_0 = ag_\sigma, \ \ \phi(\sigma) = \sigma - d\sigma^2 \ \text{ and } \ \sigma(\phi) = \frac{1-\sqrt{1-4d\phi}}{2d}, \tag{13}$$

with $d = bg_\sigma/aM_N$. This satisfies the condition: $\sigma \to 0$ in the limit $\phi \to 0$. The non-linear potential is thus calculated

$$U_s(\phi) = \frac{m_\sigma^2}{2}\left(\frac{\sigma(\phi)-\phi}{d}\right),$$
$$= \frac{m_\sigma^2}{2}\phi^2 + g_\sigma r\left(\frac{m_\sigma^2}{M_N}\right)\phi^3 + \frac{5}{2}g_\sigma^2 r^2\left(\frac{m_\sigma}{M_N}\right)^2\phi^4 + \mathcal{O}(g_\sigma^3), \qquad (14)$$

where $r = b/a$.

Table II. Parameters a, b, κ and λ.

	a	b	κ(fm^{-1})	λ
SW0	1.01	0.215	19.2	78.8
SW5	1.02	0.497	39.0	327
HO	0.687	0.245	42.3	384
BM	0.998	0.435	35.1	264

The standard form of the non-linear scalar potential is usually given by

$$U_s(\phi) = \frac{1}{2}m_\sigma^2\phi^2 + \frac{\kappa}{6}\phi^3 + \frac{\lambda}{24}\phi^4. \quad (15)$$

In more sophisticated version of QHD, [12] inspired by modern methods of effective field theory, many other terms of meson-meson and meson-nucleon couplings are considered. However, we here focus on only the parameters κ and λ to make our discussion simple. It is well known that the non-linear scalar potential, Eq.(15), is practically indispensable to reproduce the bulk properties of finite nuclei and nuclear matter in RMF.

We can estimate the parameters, κ and λ, in QMC by comparing Eqs.(14) and (15). In Table II the parameters, a, b, κ and λ, are presented. The QMC model leads to the non-linear potential with $\kappa \sim 20-40$ (fm^{-1}) and $\lambda \sim 80-400$. Since QMC predicts that both a and b are *always positive*, [1),9)] we can expect that the quark substructure of the in-medium nucleon provides a non-linear potential with *positive* κ and *positive* λ in the QHD-type mean-field model.

Using those parameters, we can draw the non-linear scalar potential generated by QMC, which is illustrated in Fig. 1. The non-linear potentials, which have been used in various relativistic mean-field models, [11)-15)] are also presented in Fig. 2. From Fig. 1 we can see that the quark models lead to the similar non-linear potentials, in spite of the big difference in the confinement mechanism. On the other hand, the non-linear potentials in RMF show quite different behaviors. The potentials in NLB [12] and PM3 [13] are relatively close to those produced by QMC.

In summary, we have calculated the properties of nuclear matter using QMC with various quark models for the nucleon. Then, we have performed a re-definition of the scalar field in matter and transformed QMC to a QHD-type model with a non-linear scalar potential. The QMC model gives $\kappa \sim 20-40$ (fm^{-1}) and $\lambda \sim 80-400$ for the non-linear scalar potential. The shapes of the potentials generated by the quark models are very similar to one another, although the confinement mechanism is quite different in each model. On the contrary, the parameters, κ and λ, phenomenologically determined in RMF take various values and the potentials in RMF are quite different from one another for large $|\phi|$. In general, the phenomenological potential may consist of the part, which is caused by the quark substructure of the nucleon, and inherent self-couplings of the scalar field in matter. It is very intriguing

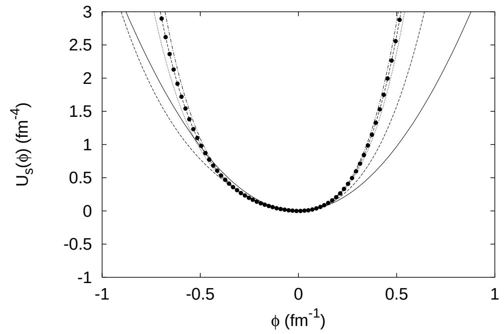

Fig. 1. Non-linear scalar potentials generated by QMC. The solid curve shows $U_s = \frac{m_\sigma^2}{2}\phi^2$. The dashed curve (with solid circles) is for SW with $\beta = 0(0.5)$, while the result of BM is shown by the dotted curve. The dot-dashed curve is for HO.

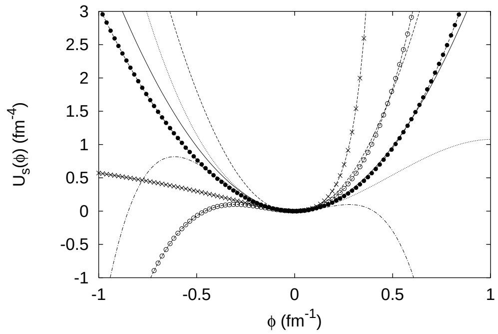

Fig. 2. Non-linear scalar potentials in RMF. The solid curve shows $U_s = \frac{m_\sigma^2}{2}\phi^2$. The dashed curve with open (solid) circles is for G2 (NLB),[12] while the dashed one without any marks is for PM3.[13] The dashed curve with crosses is for ZM.[11] The potentials in TM1[14] and NL1[15] are respectively shown by the dotted and dot-dashed curves.

if the potential due to the internal structure of the nucleon could be inferred by analyzing experimental data in the future.

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